PROMOTING THE HERITAGE OF THE CITY OF SAN CRISTOBAL DE LA LAGUNA THROUGH A TEMPORAL LINK WITH A 16TH CENTURY MAP

Fernando Pérez Nava\textsuperscript{ab}*, Isabel Sánchez Berri\textsuperscript{a}*, Alejandro Pérez Nava\textsuperscript{a}*, Virginia Gutiérrez Rodríguez\textsuperscript{b}*, Jesús Pérez Morera\textsuperscript{c}

\textsuperscript{a} Departamento de Ingeniería Informática y Sistemas, Escuela Superior de Ingeniería y Tecnología, 38203, La Laguna, Spain. fdoperez@ull.edu.es; isanchez@ull.edu.es; aperez@ull.edu.es

\textsuperscript{b} Departamento de Psicología, Facultad de Psicología. 38200, La Laguna, Spain. vgutier@ull.edu.es

\textsuperscript{c} Departamento de Historia del Arte y Filosofía, Facultad de Humanidades. 38200, La Laguna, Spain. jperezmo@ull.edu.es

Highlights:

- This study investigates the issue of georeferencing the first historic map (16th century) of San Cristóbal de La Laguna and the possibility to geolocate current city addresses.
- By using the natural interpolation method, the georeferencing errors were diminished below 4 m in most of the historic city.
- A user-friendly web application has been designed which precisely locates current places in the old map providing a valuable tool for the promotion of the heritage of the city.

Abstract:

There is an increasing interest in the conservation of historical cities since they provide a link to the roots of their communities and bring their inhabitants cultural and economic growth. In this paper, the authors present an approach to promote the knowledge of the UNESCO World Heritage city of San Cristóbal de La Laguna in the Canary Islands, Spain. The city was founded in 1496 and has a unique value due to its urban design. This work introduces a web application that allows a user to locate places and addresses of the current city in its first known map authored by Leonardo Torriani in the 16th century. To build this application we have georeferenced the ancient map in the current cartography. The georeferencing process needs the identification of homologous ground control points in the coordinate systems of both the old map and the current cartography, as well as the definition of a transformation between them. Best results were obtained with the non-parametric natural transformation interpolation, leading to a global mean error of 4.9 m reduced to 3.2 m in the historical city centre. To provide a fast response to the user of the web application, a technique to precompute offline the natural transformation is presented. The web application has a simple front-end where the user enters the current city address in a form. This activates a query to obtain the geographical coordinates of the address that are transformed into map coordinates using the pre-computed transformation. These map coordinates are used by a map viewer in the front end that locates the user address in the ancient map. To test the performance of the web application, the load of the system has been analysed obtaining a latency of 1.4 s in 50 concurrent users. Results show that the web application provides accurate results in the historical centre while offering satisfactory response times.

Keywords: historical map; cultural heritage; georeferencing; geoportal; geolocation

Resumen:

En la actualidad hay un interés creciente en la conservación de las ciudades históricas, ya que proporcionan un vínculo con las raíces de sus comunidades y aportan beneficios culturales y económicos a sus habitantes. En este artículo presentamos una aproximación para promover el conocimiento de la ciudad Patrimonio de la Humanidad de San Cristóbal de La Laguna en las Islas Canarias, España. La ciudad fue fundada en 1496 y tiene un valor único debido a su planificación urbana. Para promover su conocimiento, presentamos una aplicación web que permite a un usuario ubicar lugares y direcciones de la ciudad actual en el primer mapa conocido realizado por Leonardo Torriani en el siglo XVI. Para implementar esta aplicación se ha georeferenciado el mapa del siglo XVI en la cartografía actual. El proceso de georeferenciación necesita la identificación de puntos de apoyo terrestre homólogos en los sistemas de coordenadas del mapa antiguo y las coordenadas geográficas actuales, junto con la definición de una transformación entre ambos. Los mejores resultados se obtuvieron con una transformación mediante interpolación natural proporcionando un error medio global de 4.9 m que se reduce a 3.2 m en el centro histórico. Para generar una
PROMOTING THE HERITAGE OF THE CITY OF SAN CRISTÓBAL DE LA LAGUNA THROUGH A TEMPORAL LINK WITH A 16th CENTURY MAP

1. Introduction
The city of San Cristóbal de La Laguna (popularly known as La Laguna) is located on the island of Tenerife in Spain. Its uniqueness lies in the fact that it is the first example of a non-fortified colonial city and its urban design on a reticular arrangement. It represents the immediate predecessor of the cities founded in America after the conquest such as Old Havana, Lima, and Cartagena, where houses and streets look similar. Founded in 1486, the historical centre of the city was declared a World Heritage Site in 1999 (UNESCO, 1999).

In this paper, we present a web application that belongs to a research project which aims to develop an immersive experience of the city of La Laguna in the 16th century. The work presented here focuses on an ancient map of the city, made by the engineer Leonardo Torriani in 1588. This experience has the form of a Virtual Reality (VR) application where the user takes the role of the engineer and moves through a 3D reconstruction of the city to gather information to build this ancient map. To make this possible, he has to interact with several characters that represent the inhabitants of the city at that time and visit its main places. The use of VR applications for promoting heritage is a well-established trend worldwide (Bekele et al., 2018; Soto-Martín et al., 2020). Several studies demonstrate that the use of new and combined media enhances how culture is experienced. The goal of this project is that the VR application can significantly contribute to the documentation, conservation, and digital presentation of the historical heritage of the city and its dissemination to society through education, tourism, or research.

The use of historical maps is a research area that has concentrated on several topics like digitization (Buonora, 2009), georeferencing (Baicóchi & Lelo, 2005; Balletti, 2006; Cascón-Katchadourian et al., 2018; Livieratos, 2006; Sancho Mir et al., 2017), or web-based information systems for data publication and access (Bachiller et al., 2020; Cascón-Katchadourian et al., 2019; Santamaría-Varas & Martínez-Diez, 2019). In this work, we will focus on these topics to develop a temporal geolocation system that can be used in the VR application and as an independent web application.

This paper is divided into five parts. After the introduction, we present in Section 2 the background of the 16th century map of San Cristóbal de La Laguna. In Section 3 we show the methodologies for map digitalization, preprocessing and analysis along with the design and implementation of the web application. A discussion of the accuracy and performance of the application is presented in Section 4. Finally, the conclusions are presented in Section 5.

2. Background

2.1. Representations of the Canary Islands in the Cartography until the 16th century
The Canary Islands occupy a relevant place in geography and cartography since its inception. Traditionally identified as the mythical Fortunate Islands, they were located at the end of the known world, and, with the birth of scientific cartography in the Greco-Roman world, they also became the origin of longitudes, both in the work of Marinus de Tiro as in the Geographia of Ptolemy. After the decomposition of the Roman Empire, the European culture suffered a regression that was reflected in the cartographic elaborations, very poor and simple during the centuries corresponding to the Middle Ages. Although the knowledge of the Canaries was maintained during that time, they gradually disappeared from the cartography. There are however some maps that schematically represent the world where they appear roughly, as, for example, that of the Beatus de Valcávado, from the 10th century or the Beatus of Saint Sever, from the 11th century (Gutiérrez, 2000).

Since the last decades of the 13th century, various political and economic circumstances led some of the most powerful states in Western Europe to reaffirm their maritime policy and turned their sights towards the Atlantic. At the same time, the progressive advances that, from the second half of the 13th century and throughout the entire Late Middle Ages, were experienced in Nautical and Cosmography, made possible an incipient development of overseas companies that needed nautical charts to facilitate their expeditions. These voyages led to the rediscovery of the Canary Islands. In the mid-14th century appears the first relatively rigorous cartography of the Canary Islands both in its location and in the shape of its coasts, a few years after the rediscovery of the archipelago by western navigators (Gutiérrez, 2000). Thus, the islands of Lanzarote and Fuerteventura appear unequivocally named in the Dulcert nautical chart of 1339 and successive expeditions of exploration and conquest produced a profusion of these nautical charts. It will be necessary to wait until the 16th century for the islands to have sufficient entity to be described in isolation. They appear in the Descriptio Africæ (Fernandes, 1506), the Isolario by Bordoni (1534), the Islario by de Santa Cruz (1560) or in the Description and History of the Kingdom of the Canary Islands (Torriani, 1590). In this last manuscript is placed the map of the city of San Cristóbal de La Laguna that will be used in this work.

Virtual Archaeology Review, 13(26): 62-75, 2022

63
2.2. The city of San Cristóbal de La Laguna in the 16th century

Before the conquest, the area of the city of San Cristóbal de La Laguna was a communal pasture region and it was known by the aborigines as Aguere (de Abreu de Galindo, 1977). In this place they herded their cattle, seasonally, to take advantage of the fertile land and the water of a swampy lagoon (in fact, Aguere was an aboriginal toponym meaning laguna in Spanish). Aguere was the location of the most important battle of the conquest. The final victory of the Castilian troops in 1496 led to the annexation of the island of Tenerife to the Crown of Castile. Historical sources locate that event close to July 25th, the feast day of San Cristóbal, which gave the city its founding name.

After the defeat of the aborigines, the first feast of Corpus Christi was celebrated in the newly built church of Nuestra Señora de La Concepción. This event is considered to be the first foundation of the city and led to the settlement of soldiers and civilians with the first modest huts being built around the church, at some distance from the lagoon. This initial nucleus was known as the Upper Town where the disorderly settlement, without prior distribution of the land, led to an irregular layout of streets (Navarro Segura, 1999).

The definitive foundation of the city in 1500 was conceived from a new design centred on the current Adelantado’s Square, then known as the Lower Town, about 1 km to the southeast of the Upper Town. This new foundation, away from the lagoon, could have been done for health reasons. In this new nucleus, a layout with an ordered grid arose which followed the distribution of straight streets. In the long run, this new city design would serve as a model for the colonizing process that would later take place in the new American territories (Navarro Segura, 2006).

Thus, the city of La Laguna was, at the beginning of the 16th century, configured into two population centres. Over the following years, the two population centres, the Upper and Lower towns, gradually combined into one. The historical centre of the city was practically defined towards the end of the 16th century (Aznar Vallejo, 2008).

2.3. Description of the map

We will use the first detailed map of San Cristobal de La Laguna (Fig. 1) authored by the Italian engineer Leonardo Torriani (Cremona, 1559 - Lisboa, 1628). He was invited by the court of the Spanish King Felipe II as a military engineer and arrived at the Canary Islands in 1584 where he stayed until 1586 to prepare a report on the state of the fortifications of the islands. After returning to Court, he returned to the Canary Islands in 1587, where he stayed for six more years, visiting all the islands, writing reports on their fortifications, and collecting impressions and notes, which gave, as a result, his work Description and History of the Kingdom of the Canary Islands (Torriani, 1590), which remained unpublished in the convent of São Benito and the University of Coimbra until 1940. On this date, it was first published by the German studious D. J. Wöfler. The final edition will be available almost 20 years later (Torriani, 1959), edited by A. Cioranescu with an excellent introduction and numerous notes.

Figure 1: Torriani’s map of San Cristóbal de La Laguna (source: Library of the University of Coimbra).
The Description of the Canary Islands is composed of three different elements that are present in all the chapters dedicated to the islands in the archipelago. The first one is a general framework where the geographic conditions of the islands are represented. The second one is dedicated to the fortifications and the third one is dedicated to the history of the island. The map of San Cristóbal de La Laguna is present in chapter 13th of the Description. On the map (Fig. 1), we can see the main elements of the city: The irregular Upper Town in the lower part of the map on the side of the lagoon and the regular Lower Town on the upper side of the map. Around that time, in 1592, La Laguna was the main city of the island and had 952 houses and 5032 inhabitants (Cabrera, 1987) that are approximately represented on the map. The map also depicts 2 churches, 3 hermitages, 4 convents, 2 hospitals and 1 government building (Casas del Cabildo). The orientation of the map is defined by an arrow drawn on the lagoon and the north points towards the left of the map.

3. Methodology

3.1. Map digitalization

The manuscript of the Description of the Canary Islands is conserved in the Library of the University of Coimbra (Portugal). A scanning request was made to the Library at the highest resolution. The digitised image was obtained as an image file with 300 dots per inch. The original manuscript has 235 x 414 mm which led to an image with 5125 x 3240 pixels. The colour depth is 24 bits. The image is shown in Figure 1.

3.2. Map preprocessing

The map of San Cristóbal de La Laguna has two main damages. The first one is due to the overprint of the previous page of the manuscript and runs through the upper part of the map and can be seen in Figure 1 and Figure 2a. The overprint has been digitally removed by hand by cloning similar image patches from the neighbourhood of the damaged parts. Results are shown in Figure 2c.

Figure 2: Overprinting restoration: a) Damage from overprinting from the previous page; b) Overprint from the previous page; c) Damage removed.

The second one can be seen on the legend of the map as an adhesive tape that has been fixed to mend a rupture of the map (Fig 3a). This adhesive tape has been digitally removed by cloning other parts of the legend. Results are shown in Figure 3b.

Figure 3: Legend restoration: a) Legend of the map damaged with the adhesive tape; b) Restored legend.

3.3. Map analysis

From the high-resolution digitised copy of the map, we have carried out an analysis to evaluate the scale, orientation, and geometric quality of the ancient map. This evaluation is achieved by means of the georeferencing process (Baiocchi & Lelo, 2005; Balletti, 2006; Hill, 2009), which relates the internal coordinate system of a map or aerial photo image to a georeferenced coordinate system. Its usage with the help of digitisation techniques and web publication facilitates access for non-expert users to geographical, heritage, urban or environmental information of interest (Cascón-Katchadourian et al., 2018; Talich, 2020). Georeferencing is today an important area of study within the field of cartography (Long et al., 2016). It is a fundamental element in the analysis of ancient cartography (Alvares-Sanches et al., 2020; Guarducci & Tarchi, 2020; Koussoulakou et al., 2020) as it allows us to accurately reconstruct the geographical situation of maps. Its usage has allowed to revitalise and give new purpose to numerous and diverse ancient cartography that remains in archive deposits, libraries, and cultural centres.

In cartography (Cascón-Katchadourian et al., 2018), the georeferencing process begins with the identification of homologous ground control points (GCPs) in the local coordinate system of the image/map and the coordinate system of a support map (also known as reference cartography) (Dávila & Camacho, 2012). The process usually consists of establishing a set of reliable common locations which both coordinate systems, and that has been maintained over time (geographical features, outlines of city blocks, buildings, monuments, streets), thus indicating that those homologous GCPs are geographically the same.

There are some important recommendations for GCPs collection (Cajthaml, 2011). GCPs should be laid out through the whole map image (if possible); GCPs should be represented by stable well-identifiable objects, and the more GCPs collected the better input data for georeferencing.

A total of 221 GCPs were located on Torriani’s map. Most of them are placed at the corners of the city blocks which have not been subject to significant changes over time. The corresponding pairs of the old map have been located in modern cartography using the vectorised block layer of the cadastral data available from the city (Catastro, 2021) and the National Plan of Aerial Orthophotography (Plan Nacional de Ortofotografía...
Aérea, PNOA) from the city (Centro Nacional de Información Geográfica, 2021). An open-source Geographical Information System (QGIS, 2021) has been used to locate the GCPs in the ancient map and their homologous points in modern cartography. As can be seen in Figure 4, the historic city centre has been exhaustively covered including the Upper and Lower Town. A particular difficulty appears when trying to locate GCPs outside the town. In this case, we have mostly relied on the orthophoto for correspondences, though in this case, we expect much less precision.

Once the GCPs have been selected in both the old map and the modern cartography, a transformation between them has to be selected. Two main groups of transformations can be used: global, and local. In global transformation methods, the same transform is used for the whole set of points. Examples of this kind of transformation are: similarity (or Helmert), affine, polynomial, or projective transforms (G. Bitelli et al., 2009; Bitelli & Gatta, 2012; Bower, 2009; Cajthaml & Janata, 2017; Camacho et al., 2019; Manzano-Agugliaro et al., 2012; Pindozzi et al., 2016). On the other side, in local transformations, every point is transformed with its own equations that are determined by its spatial vicinity (Tucci, 2010). Examples of this kind of transformation are: natural interpolation, thin-plate splines, or inverse weighted distance interpolation.

3.3.1. Scale and orientation of the map

The map under consideration has no scale reference but it is possible to find the orientation of the map as an arrow that points to the north in the lagoon (Fig 5a). Using image processing techniques we have segmented its shape (Fig. 5b) to determine its alignment relative to the scanned image.

To determine the global scale and orientation of the map we have used the similarity transformation. However, since we expect that the parameters of the transformation to be dependent on location, we studied the variations of scaling and rotation through the map. Using the software tool Mapanalyst (Jenny & Hurni, 2011), it is possible to compute and display the local scale variation within the ancient map, coupled with the visualisation of the distorted grid of the modern cartography over the old map.

3.3.2. Planimetric accuracy

To determine the positional accuracy of the georeferencing process, we must first determine a global or local transformation between the homologous GCPs defined in the historical map and current cartography. As suggested in (Cajthaml, 2011) for the case of only a map
sheet, unknown projection and unknown dimensions, we georeference the modern map in geographic coordinates (Coordinate Reference System REGCAN95, EPSG:4081). For completeness, we have also studied the behaviour of the georeferencing process in the projected UTM coordinates REGCAN95 UTM 28N, EPSG:4083.

To determine a global transformation with GCPs, its parameters will be determined usually by the least-squares method (LSM). If the set of points has more than the minimal number of GCPs to uniquely determine the global transform, residuals will be generated. By examining the residuals, a quantitative study of the fitting errors can be done. We have used the mean of the residuals as the positional accuracy estimate. To avoid overfitting and error underestimation, each one of the selected GCPs was left out from the computation of the transformation and its error was computed and averaged for all the GCPs.

Since we expect some outliers in the assignment of the GCPs due to the imprecision of the matching process, we have used the R-estimator Sample Consensus (MSAC) for a robust estimation of the parametric transformation (Fernández et al., 2021; Fischler & Bolles, 1981; Torr & Zisserman, 2000). The 90th percentile of the error distance without considering outlier points has been used as a parameter for the maximum distance in the MSAC estimator.

The robust error is obtained from the subset of GCPs detected as inliers by the MSAC estimator. To avoid robust error underestimation, each one of the GCPs was left out from the computation of the MSAC estimator and its error was computed in case of being an inlier. The mean of the error of all inliers is the measure of the robust error.

For the local transformations, every point is transformed with its own local equations that are determined by a spatial neighbour. Usually, the minimum number of GCPs is selected and therefore no residuals are generated. In our case, we have selected natural neighbour interpolation. This method finds the closest subset of input samples to a query point and applies weights to them based on proportionate areas to interpolate a value (Sibson, 1981). The interpolated surface passes through all the input samples and is smooth everywhere except at the locations of the input samples. Since this method gives the exact values for all the considered GCPs the error is zero. Then, to obtain a pixel estimation error for this method, we have left one of the GCPs out, estimated the transform for the rest and estimated the transformation error from the removed point. Repeating this for all the GCPs and computing the mean of the errors we obtain the fitting error. The GCPs whose errors were greater than the 90th percentile of the fitting error were considered as outliers and removed to compute also a robust error estimation.

3.3.3. Local transformation optimization

A major drawback of local transformation methods is that they are more complex to compute due to their point neighbourhood dependence and, therefore, are slower than global methods. Since our goal is to develop a web application it is important to reduce to the minimum the computational load and response time of the server. This is done by decomposing the transformation with natural interpolation between the modern cartography and the old map in two steps: first, a global similarity transform is estimated between the modern cartography and the old map. This global transform allows us to work with current cartography in the old map coordinates. Then, we try to predict the points in the old map using these transformed points using natural interpolation. The process is seen in Figure 6. Since the natural interpolation is invariant to rigid transforms, we obtain the same result as with the usual natural interpolation.

![Figure 6: Two-step interpolation process.](Image)

The advantage of this decomposition relies on the fact that the second step can be stored as two precomputed images of the size of the old map, one for the row coordinates $Tr_r$ and the other for the column coordinates $Tr_c$. This is possible now since both the origin of the transformation and its destiny are in old map coordinates.

A potential drawback of this approach arises when we store in the images $Tr_r$ and $Tr_c$ the coordinates of the predicted points in the image of the old map. If the old map image is large, each pixel may have a big value that would not allow us to use common image formats. To solve this problem, as we expect that the errors of the global prediction are small, we subtract to each pixel in the row image the value of its row and also we subtract to each pixel in the column image the value of its column. This makes that the range of the two images is small. Notice that these images can be interpreted as the part of the transformation not accounted with the similarity counterpart.

Now, for a point with coordinates $(x, y)$ in the current cartography, if we want to compute the corresponding point in the old map, we first transform it using the estimated similarity transformation (4 multiplications and two additions) obtaining a point with coordinates $(row, column)$ in the old map. The predicted coordinates are shown in Eq. 1:

$$Predicted\ row = row + Tr_r(row, column)$$
$$Predicted\ column = column + Tr_c(row, column)$$

(1)

Needing only two memory accesses and two more additions.

3.3.4. Map tiling

To distribute the geolocated image on the web, a map service must be used. To standardize this kind of application, the Open Geospatial Consortium (OGC) developed the Web Map Service (WMS) recommendation (De la Beaujardiere, 2006). This standard provides a simple HTTP interface for requesting geo-referenced map images from one or more distributed geospatial databases. It was designed so that clients can request arbitrary sized map images to the server, superposing multiple layers covering an arbitrary geographic area. However, this
flexibility reduces the potential to cache map images because the probability of receiving two exact map requests is very low. This involves a very time-consuming and computationally expensive generation process that negatively affects service scalability and users’ quality of service.

A common approach to improve the catchability of requests is to divide the map into a discrete set of images, called tiles, and restrict user requests to that set (García et al., 2012). The benefit of tiled map services is that map image tiles can be cached at any intermediate location between the client and the server, reducing the latency associated with the image generation process. Tile caches are usually deployed server-side, serving map image tiles concurrently to multiple users. Several specifications have been developed to standardize this process. The Open-Source Geospatial Foundation (OSGeo) developed the WMS Tile Caching (usually known as WMS-C) proposal. Later, the OGC released the Web Map Tile Service Standard (WMTS) inspired by the former. The properties of tiled maps that require standards include the tile size, the numbering of zoom levels, the projection to use, the way individual tiles are identified, and the method for requesting them.

In our web application, we will use the de-facto OpenStreetMap XYZ standard where each tile is stored as an image with a size of 256×256 pixels. These images are arranged so that each zoom level is a directory, each column is a subdirectory, and each tile in that column is individually stored. The filename (URL) format is defined as /zoom_level/x/y.png. The tiles can be easily generated with the open-source (GDAL, 2020) library. In our application, we clipped the original image to the historic centre obtaining an image with 3584×3584 pixels. Then, five zoom levels were defined to be available to the user.

3.4. Design and implementation of the Web service

3.4.1. Overview of the web application

A very easy-to-use web application, aimed at the general public, has been developed, allowing the combination of current cartographic information with the historical map of the city. This facilitates the integration of the georeferenced map with other information sources for its use in a dynamic user experience. The web application is an example of a tool for the dissemination of Torriani’s map and to give visibility to the evolution of the city. Through the application interface, the user can provide an address in the current street map of San Cristóbal de La Laguna and its location in the 16th century map is shown by the web application with an overlaying coloured marker. A prototype of the application can be accessed through the URL http://torriani.iaas.ull.es/.

3.4.2. Design and implementation of the web application

To describe the web application, we will show the processing that is made on the server and client side.

In the front-end of the application, there are two well-differentiated elements, a form to fill in the current city address and activate the query on the top and the map viewer on the bottom (Fig. 7). The address input form has been implemented with jQuery and (Materialize, 2021), a user interface component library created with CSS, JavaScript, and HTML. The map viewer has been implemented using the javascript library OpenLayers, which provides mechanisms for loading and interacting on the map as well as adding markers.

Some design decisions have been made to benefit the usability of the web application. To prevent errors at the client-side, information on incomplete data is provided to help the users to avoid mistakes. Warnings are displayed regarding addresses outside the historic centre of the city. A responsive design has been followed that allows the user to easily zoom in and pan through the map. Also, to ease the user to locate the current city on the old map, current street names have been inserted through the 16th century map.

To determine the position in which the marker in the old map should be located, it is necessary to map the address provided by the user in the current coordinates to the coordinates on Torriani’s map. This process is solved by obtaining the geographic coordinates of the address that will be transformed to the correct position on the map. In our case, the (Cartociudad, 2021) API is used to obtain these coordinates. The choice of this API is due to its wide coverage and high precision. CartoCiudad is a national continuous urban and interurban road network database, generated from official data. Its main data source is the road network Geographical Reference Information on Transport Networks (RT) of the Spanish Institute Geográfico Nacional (IGN), combined with data from other agencies: the Dirección General del Catastro (DGC), the Instituto Nacional de Estadística (INE) and the Correos Group. In addition to this, some local government entities have collaborated on its preparation and maintenance (Mas et al., 2013).

The position of the marker on the old map is determined by computing the predicted coordinates (Eq. 1) for the geographic coordinates provided by CartoCiudad. These calculations require access to each of the precomputed images Tr and Tc in Step 2 of the map transformation in Section 3.3.3. All this process is carried out on the server through a RESTful API. This design avoids the net traffic and consequent loss of performance that would be generated if the Tr and Tc images were downloaded by each client. Finally, a nginx web server has been used to act as a proxy that redirects the traffic towards the requests to the web application or to the API for obtaining coordinates on the map. The API is implemented on a node.js and Express server. The full system is shown in Figure 8.
ancient map into modern cartography. The estimated scaling factor using projected coordinates is $0.4502 \pm 0.0033$ which also corresponds to an image pixel size in meters. The estimated rotation angle of the image is $97.0434 \pm 0.6739$ counterclockwise. The estimation intervals are obtained from the inliers of the MSAC estimator presented in Section 3.3.2. Each one of the GCPs was left out from the computation of the MSAC estimator and the scaling and rotation angle were computed in case of being an inlier. The mean and standard deviation of all the estimations were used to build the intervals.

The scaling factor and the scanning resolution allow us to estimate the scale of the map as 1:5322. The estimated angle of the north arrow relative to the scanned map image is estimated as $0.1096$ from the segmented image in Figure 5. Therefore, the estimated rotation angle of the map is $96.9338$ counterclockwise.

We studied the variations of scaling and rotation through the map, which may arise in the process of making the map in the 16th century or posterior deformations of the analogical support. Results from the study of map deformation show that scale variation appears quite constant over the historic centre as seen in Figure 9. The same happens for the angular variation. The lack of references outside the historic centre increases the deformations outside the city limits. Figure 9 also shows the inliers of the transformation (in red). They are concentrated inside the borders of the city while outliers (in green) are distributed mainly outside the city.

**Figure 8:** Web application architecture.

**Figure 9:** Deformation analysis of the map.
4.2. Spatial map accuracy
To determine the optimal transformation, map accuracy has been studied for both parametric and non-parametric transforms using robust estimation. Parametric transforms are simple and fast but their adjustment is worse when deformations are complex. On the other side, non-parametric transformations offer better adjustment at the cost of higher computational costs.

4.2.1. Parametric transformations
To minimize the fitting errors, several global transformations have been studied (Musin, 1991). Results for five global transformations are shown in Table 1. The error in the table is computed as the mean of the distances between the transformed GCPs in modern cartography and its corresponding pair in the map and is measured in pixels of the old map image and meters. To compute the error in meters, we used the scaling factor arising from the similarity transformation in Section 4.1.

As expected, results show that complex transformations achieve lower errors since there are more parameters to fit. The robust error varies from 14.0 m in the similarity transformation with 4 parameters to 6.1 m for a degree 3 polynomial with 20 parameters. The use of a dense distribution of the GCPs in the historic centre controls the non-linearities arising from the projective and polynomial transformations.

The change to projected coordinates for the GCPs only significantly improved the transformation error in the similarity case from 14.0 m to 8.3 m (Table 2). For the other transforms in Table 1, the difference was below one pixel. This indicates that the use of geographic coordinates introduces a non-rigid transformation that is greatly reduced in the projected coordinates.

The error histogram and the position of the transformed old map over the modern cartography for the projective transformation are shown in Figures 10 and 11.

<table>
<thead>
<tr>
<th>Transform</th>
<th>Error (pixels)</th>
<th>Robust Error (pixels)</th>
<th>Error (m)</th>
<th>Robust Error (m)</th>
<th>Number of params.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Similarity</td>
<td>51.3</td>
<td>31.0</td>
<td>23.1</td>
<td>14.0</td>
<td>4</td>
</tr>
<tr>
<td>Affine</td>
<td>27.0</td>
<td>17.7</td>
<td>12.2</td>
<td>8.0</td>
<td>6</td>
</tr>
<tr>
<td>Projective</td>
<td>25.1</td>
<td>16.7</td>
<td>11.3</td>
<td>7.5</td>
<td>8</td>
</tr>
<tr>
<td>Polynomial degree 2</td>
<td>21.6</td>
<td>15.7</td>
<td>9.7</td>
<td>7.1</td>
<td>12</td>
</tr>
<tr>
<td>Polynomial degree 3</td>
<td>16.3</td>
<td>13.5</td>
<td>7.3</td>
<td>6.1</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 2: Pixel errors for different transforms in pixels and meters in projected coordinates for the similarity transform.

<table>
<thead>
<tr>
<th>Transform</th>
<th>Error (pixels)</th>
<th>Robust Error (pixels)</th>
<th>Error (m)</th>
<th>Robust Error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Similarity</td>
<td>25.4</td>
<td>18.5</td>
<td>17.4</td>
<td>8.3</td>
</tr>
</tbody>
</table>

Figure 10: Histogram of the errors in m for the projective case.

Figure 11: Torriani’s map overlaid over modern cartography.
As we can see in Table 1 and Figure 11, many fitting errors in the historic centre are within the width of a typical street which is quite remarkable considering the age of the map. Large errors are found as we leave the historic centre and reach the city limits in all directions.

4.2.2. Non-parametric transformations

To improve the geolocation in the historic centre we turned to a local transformation to obtain a better fit. In this case, we have used the natural neighbour interpolation. To obtain a pixel estimation error for this method, we have left one of the GCPs out, estimated the transform for the rest and estimated the error from the removed point. Repeating this for all the GCPs and computing the mean of the errors we obtain a fitting error of 4.9 m (Table 3). The robust error, as defined in Section 3.3.2, is equal to 3.2 m, a measure that is acceptable for the web application.

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Error (pixels)</th>
<th>Robust error (pixels)</th>
<th>Error (m)</th>
<th>Robust error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural</td>
<td>10.9</td>
<td>7.0</td>
<td>4.9</td>
<td>3.2</td>
</tr>
</tbody>
</table>

The error histogram for natural interpolation is shown in Figure 12. Note that is much more concentrated around 0 than the histogram of Figure 10 for the projective case.

The result of the transform of the modern blocks overlaid over Torriani’s map for natural interpolation is shown in Figure 13. Results show a very good fit over the historic centre. The use of projected coordinates in the GCPs did not improve the results in Table 3 significantly. The error difference was below 0.1 pixels.

The results outside the historic centre are worse and it is very difficult to improve them for several reasons. The main one is that there are no matching points in the map outside the city because there Torriani represents generic fields and the lagoon in the map was desiccated in 1839. Also, since the outliers of the transform are located in the border of the city (Fig. 9) it is not possible to obtain a good extrapolation outside the city limits. Finally, Torriani sacrifices geometric precision for site information in some borders of the map. For example, he considers that it is important to show how the mountains delimit the city meadow on the east. This leads to considerable geometric errors in the top left corner of the map.

4.3. Performance of the web application

To test the performance of the web application, the behaviour of a typical internet user was recorded using (Fiddler, 2021). A scenario was defined using (Octoperf, 2021) in which 50 concurrent users accessed the web application. The number of users grew linearly in the first five minutes and then remained constant for another five minutes.

The images for the row and column transformation maps in Section 3.3.3 were computed and are shown in Figure 14. In the period under study, the average response time to compute the transform from geographical coordinates to map coordinates was 0.108 s which validates our approach. We also measured the performance of the whole web application for a typical application user obtaining the results in Table 4.

<table>
<thead>
<tr>
<th>Number of hits</th>
<th>Average response time (s)</th>
<th>Latency standard deviation (s)</th>
<th>Received bytes rate (Mb/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>72640</td>
<td>1.4</td>
<td>1.1</td>
<td>13.6 Mb</td>
</tr>
</tbody>
</table>

Results in Table 4 and Figure 15 show an adequate average response time that remains nearly constant and is not degraded by the increase in the number of users.

5. Conclusions

The interest in the conservation of historic cities has generated the need to develop tools to promote its heritage so that it can become part of our collective knowledge and thus last over time. In this paper,
we have presented an approach to spread the knowledge of the UNESCO’s World Heritage Site city of San Cristóbal de La Laguna in Spain. It has a unique value due to the original conception of its urban planning which was used as a template for the colonial cities founded in America.

In our approach, a web application has been designed that allows a user to locate places of the current city in the first known map of the city by the engineer Leonardo Torriani in the 16th century. This study investigated the issue of georeferencing this historical map and the possibility to geolocate the current city addresses over it.

The georeferencing process began by digitally restoring some defects of the ancient map that was scanned by the Library of the University of Coimbra. Then, a set of 221 corresponding GCPs were located in the old map and modern cartography. Due to the map composition, most of them were at the historic centre and a few of them were located outside the city. Using these GCPs, a set of parametric transforms were studied using the robust MSAC estimator. Our results show that matching inliers are distributed inside the city while outliers are defined over the border and outside the city. Minimum errors were obtained with the parametric transformation defined by a polynomial of degree 3 with a mean value of 7.3 m. To improve this result, we used the non-parametric natural interpolation method. In this case, the mean of the georeferencing errors was diminished to 4.9 m on the whole map and 3.2 m in the historic city. The estimated scale of the map is 1:5322 and its orientation $96^\circ.9338$ counterclockwise. Results show that precise geolocation of the old map can be achieved. This is a remarkable result considering the time difference between the map and modern cartography. An analysis of the geolocation process outside the city was also presented and its difficulties were discussed.

After the geolocation process was completed, we presented our web application. A simple front end has been designed where the user fills in the current city address in a form. This activates a query using the CartoCiudad API to obtain the geographical coordinates of the address that are transformed to map coordinates using the natural transformation. These coordinates are used by a map viewer in the front end to locate the user address in the ancient map. To improve the response time, the natural transformation is precomputed as two images of the size of the map. This allows the
application to obtain the coordinates transformed with simple memory access.

To test the performance of the web application the behaviour of a typical internet user was studied, and the load of the system has been analyzed. The system obtained an average response time of 1.4 s for 50 concurrent users. Therefore, the web application provides accurate results in the historical centre while providing satisfactory response times.

Future work includes the use of the developed tools to geolocate the address in more historic maps of the city so that the user can see the evolution of the city over nearly five hundred years.

Acknowledgements
This work was supported by the Fundación Cajacanarias and Fundación Bancaria “La Caixa” under “Patrimonio Cultural en Canarias: del legado aborigen a la nueva sociedad canaria”.

References


**Centro Nacional de Información Geográfica.** (2021, May 28). *Centro de Descargas del CNIG (IGN).* http://centrodescargas.cnig.es


