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Deteriorating inventory model for quadratically time varying demand with partial backlogging

Vinod Kumar Mishra

Department of Computer Science & Engineering, B. T. Kumaon Institute of Technology, Dwarahat, Almora, -263653, Uttarakhand, INDIA,Email- vkmishra2005@gmail.com

Abstract: In this paper, a deterministic inventory model is developed for instantaneous deteriorating items in which shortages are allowed and partially backlogged. Deterioration rate is constant, demand rate is quadratic function of time and holding cost is linear function of time, backlogging rate is variable and depends on the length of the next replenishment. The model is solved analytically by minimizing the total inventory cost. This inventory model can also use as an inventory model for linear as well as constant demand rate by very small change in the incremental factor of the quadratic function. Numerical examples are provided to illustrate the solution and application of the model.

Keywords: Inventory; instantaneous deteriorating ítems; shortages; quadratic time varying demand; partial backlogging; time dependent holding cost.

1. Introduction

One of the most unrealistic assumptions in traditional inventory model was that items preserved their physical characteristics while they were kept stored in inventory and demand rate & holding cost did not vary with respect to time. However, the deteriorating items are subject to a continuous loss in their masses or utility throughout their life time due to decay, damage, spoilage and plenty of other reasons and demand rate & holding cost vary according to time due to change of economy with respect to time. Owing to this fact controlling and maintaining inventory of deteriorating items becomes a challenging problem for decision makers.

Inventory of deteriorating items first studied by Whitin (1957), he considered the deterioration of fashion goods at the end of prescribed storage period. In (1963) Ghare and Schrader extended the classical EOQ formula with exponential decay of inventory due to deterioration and gave a mathematical model of inventory of deteriorating items. Chang and Dye (1999) developed an inventory model with time varying demand and partial backlogging. Goyal and Giri (2001) gave recent trends of modeling in deteriorating items inventory. They classified inventory models on the basis of demand variations and various other conditions or constraints. Ouyang et al. (2005) developed an inventory model for deteriorating items with exponential declining demand and partial backlogging.

Dye et al. (2007) developed an inventory model to find an optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging. They assume that a fraction of customers who backlog their orders increases exponentially as the waiting time decreases for the next replenishment. Alamri and Balkhi (2007) studied the effects of learning and forgetting on the optimal production lot size for deteriorating items with time varying demand and deterioration rates. Teng et al. (2007) gave a comparison between two pricing and lot-sizing models with partial backlogging and deteriorated items. Dye (2007) gave an inventory model to determining optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging and deterministic inventory model for deteriorating items with capacity constraint and time-proportional backlogging rate. Roy (2008) developed an inventory model for deteriorating items with time varying holding cost and demand is price dependent.

Pareek et al. (2009) developed a deterministic inventory model for deteriorating items with salvage value and shortages. Skouri et al. (2009) developed an inventory models with ramp type demand rate, partial backlogging and Weibell`s deterioration rate. He et al. (2010) gave an optimal production inventory model for deteriorating item with multiple market demand. Mandal (2010) gave an EOQ inventory model for Weibull distributed deteriorating items under ramp type demand and shortages. Chang et al. (2010) gave an optimal replenishment policy for non instantaneous deteriorating items with stock dependent demand. Hung (2011) gave an inventory model with generalized type demand, deterioration and backorder rates.

Mishra and Singh (2011) developed deteriorating inventory model for time dependent demand and holding cost with partial backlogging. They made Abad (1996, 2001) and Mishra and Singh (2010) more realistic and applicable in practice. Begum et al. (2012) gave an inventory model for non instantaneous deteriorating items with quadratic demand and partial backlogging.

In classical inventory models the demand rate and holding cost has assumed to be constant but in some realistic situation of inventory the demand and holding cost both are vary according to the time i.e. Time will play an important role in the inventory system. In this paper I made the model of Abad (1996,2001) and Mishra and Singh (2010, 2011) more realistic and applicable by considering demand as a quadratic function of time and holding cost as linear function of time with constant rate of deterioration. The assumption of quadratic demand is useful for the items whose demand increases very rapidly, such as newly launched products in the market .Shortages are allowed and partially backlogged; backlogging rate is variable and is dependent on the length of the next replenishment. This inventory system can be use as an inventory system for linear as well as constant demand rate by putting the value of incremental factor of quadratic function, c=0 and b= c=0, respectively.

The assumptions and notations of the model are introduced in the next section. The mathematical model and solution procedure is derived in section 3 and 4 respectively and numerical and graphical analysis is presented in section 5. The article ends with some conclusion and scope of future research.

2. Assumption and Notations

The mathematical model of the inventory system is based on the following notations and assumptions.

Notations

- A the ordering cost per order.
- C_1 the purchase cost per unit.
- θ the deterioration rate.
- h(t) the inventory holding cost per unit per time unit.
- π_b the backordered cost per unit short per time unit.
- π_L the cost of lost sales per unit.
- t_1 the time at which the inventory level reaches zero, $t_1 \geq 0$

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- t_2 the length of period during which shortages are allowed, $t_2 \geq 0$
- T $(= t_1 + t_2)$ the length of cycle time
- IM the maximum inventory level during [0, T].
- IB the maximum inventory level during shortage period.
- Q (= IM + IB) the order quantity during a cycle of length T.
- I1 (t) the level of positive inventory at time t, $0 \leq t \leq t_1$
- I2 (t) the level of negative inventory at time t, $t_1 \leq t \leq t_1 + t_2$
- TC (t_1, t_2) the total cost per time unit.

Assumptions

- The demand rate is time dependent that is if 'a' is fix fraction of demand and 'b' and 'c' are the fraction of demand which is vary with time then demand function is $f(t) = a + b\,t + ct^2$, where a>0 ,b>0 c>0.
- Holding cost is linear function of time $h(t) = \alpha + \beta t$, $\alpha \geq 0$, $\beta \geq 0$.
- Shortages are allowed and partially backlogged.
- The lead time is zero.
- The replenishment rate is infinite.
- The planning horizon is infinite.
- The deterioration rate is constant.
- During stock out period, the backlogging rate is variable and is dependent on the length of the waiting time for next replenishment. So that the backlogging rate for negative inventory is, B (t) = $\frac{1}{1+\delta(T-t)}$ where δ is backlogging parameter and (T-t) is waiting time during ($t_1 \leq t \leq T$).

3. Mathematical Model

The rate of change of inventory during positive stock period $[0,t_1]$ and shortage period $[t_1,T]$ is governed by the differential equations

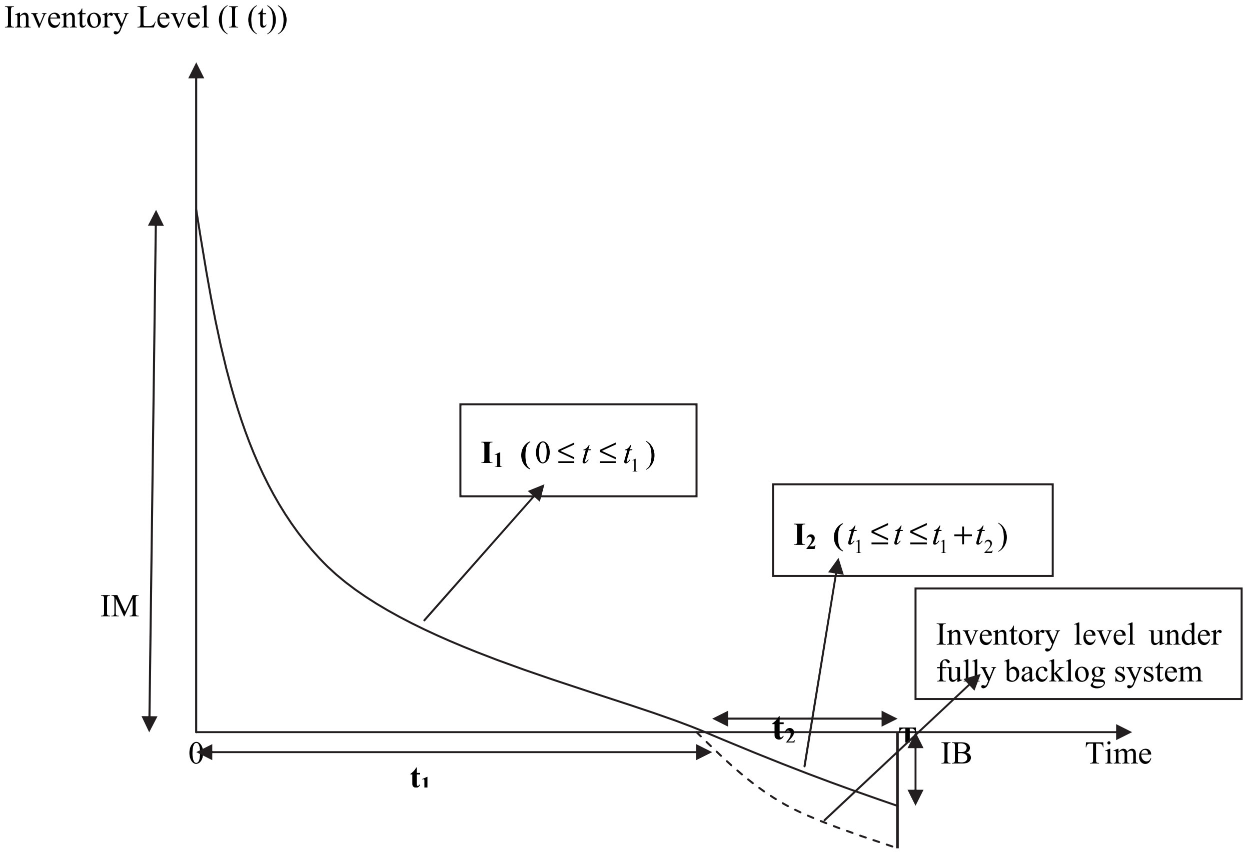
$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -\left(a + bt + ct^2\right) \qquad 0 \leq t \leq t_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} = \frac{-(a + bt + ct^2)}{1+\delta(T-t)} \qquad t_1 \leq t \leq T \tag{2}$$

With boundary condition

$I_1(t) = I_2(t) = 0$ at $t = t_1$ and $I_1(t) = IM$ at $t=0$

Figure -1-: (Graphical Representation of Inventory System)



4. Analytical Solution

Case I: Inventory level without shortage

During the period [0, t_1], the inventory depletes due to the deterioration and demand. Hence, the inventory level at any time during [0, t_1] is described by differential equation

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -(a + bt + ct^2) \qquad 0 \le t \le t_1 \tag{3}$$

With the boundary condition $I_1(t_1) = 0$ at $t = t_1$

The solution of equation (3) is

$$I_1(t) = \left[-\frac{a}{\theta} - \frac{b}{\theta}(t - \frac{1}{\theta}) - \frac{c}{\theta}(t^2 - \frac{2t}{\theta} + \frac{2}{\theta^2}) + e^{\theta\left(t_1 - t\right)} \left(\frac{a}{\theta} + \frac{b}{\theta}(t1 - \frac{1}{\theta}) + \frac{c}{\theta}(t_1{}^2 - \frac{2t_1}{\theta} + \frac{2}{\theta^2}) \right) \right]$$

$$; \quad 0 \le t \le t_1 \tag{4}$$

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Case II: Inventory level with shortage

During the interval $[t_1, t_1 + t_2]$ the inventory level depends on demand and a fraction of demand is backlogged. The state of inventory during $[t_1, t_1 + t_2]$ can be represented by the differential equation

$$\frac{dI_2(t)}{dt} = \frac{-(a+bt+ct^2)}{1+\delta(t_1+t_2-t)}; t_1 \le t \le t_1 + t_2 \quad (5)$$

With the boundary condition $I_2(t_1) = 0$ at $t=t_1$

The Solution of equation (5) is

$$I_2(t) = \begin{bmatrix} \left(\frac{a}{\delta} + \frac{b}{\delta^2} + \frac{bt_1}{\delta} + \frac{bt_2}{\delta} + \frac{c}{\delta^3} + \frac{2ct_1}{\delta^2} + \frac{2ct_2}{\delta^2} + \frac{ct_1^2}{\delta} + \frac{ct_2^2}{\delta} + \frac{2ct_1t_2}{\delta}\right) \\ \log\left(\frac{1+\delta(t_1+t_2-t)}{1+\delta t_2}\right) + \frac{b(t-t_1)}{\delta} + \frac{c\left(\frac{t^2}{2} - \frac{3t_1^2}{2} + t_1t + tt_2 - t_1t_2\right)}{\delta} + c\frac{(t-t_1)}{\delta} \end{bmatrix} \quad (6)$$

Therefore the total cost per replenishment cycle consists of the following components:

1) Inventory holding cost per cycle;

$$IHC = \int_0^{t_1} h(t) I_1(t) dt$$

$$IHC = \int_0^{t_1} (\alpha + \beta t) I_1(t) dt$$

$$IHC = \begin{bmatrix} -\frac{1}{12\theta^5}(-12\beta b\theta + 24\alpha c\theta + 12\beta a\theta^2 - 12b\theta^2\alpha - 24c\beta e^{\theta t_1} \\ +4b\theta^4\beta t_1^3 + 3c\theta^4\beta t_1^4 - 12a\alpha\theta^3 e^{\theta t_1} - 12a\beta\theta^2 e^{\theta t_1} - 24c\alpha\theta e^{\theta t_1} \\ +12b\alpha\theta^2 e^{\theta t_1} + 12b\beta\theta e^{\theta t_1} - 12c\alpha\theta^3 t_1^2 e^{\theta t_1} - 12b\alpha\theta^3 t_1 e^{\theta t_1} \\ +24c\beta\theta t_1 e^{\theta t_1} - 12c\beta\theta^2 t_1^2 e^{\theta t_1} - 12b\beta\theta^2 t_1 e^{\theta t_1} + 24c\alpha\theta^2 t_1 e^{\theta t_1} \\ +6b\beta\theta^3 t_1^2 + 12a\alpha\theta^4 t_1 + 6b\alpha\theta^4 t_1^2 + 6a\beta\theta^4 t_1^2 + 4c\theta^4\alpha t_1^3 \\ +4c\theta^3\beta t_1^3 + 24\beta c + 12a\alpha\theta^3 + 12a\beta\theta^3 t_1) \end{bmatrix} \quad (7)$$

2) Backordered cost per cycle;

$$BC = \pi_b\left(\int_{t_1}^{t_1+t_2} -I_2(t)\, dt\right)$$

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$$BC=\frac{\pi_b}{6\delta^4}\left(\begin{array}{l} 12ct_1t_2\delta^2+6c\log\left(\frac{1}{1+\delta t_2}\right)+6a\log\left(\frac{1}{1+\delta t_2}\right)\delta^2+6b\log\left(\frac{1}{1+\delta t_2}\right)\delta \\ +6bt_2t_1\delta^3+6bt_2\delta^2+6ct_2\delta+9ct_2^2\delta^2+6at_2\delta^3+6ct_1^2t_2\delta^3+6ct_1t_2^2\delta^3 \\ +12ct_1t_2\log\left(\frac{1}{1+\delta t_2}\right)\delta^2+6b\log\left(\frac{1}{1+\delta t_2}\right)t_1\delta^2+12c\log\left(\frac{1}{1+\delta t_2}\right)t_1\delta \\ +6ct_1^2\log\left(\frac{1}{1+\delta t_2}\right)\delta^2+12c\log\left(\frac{1}{1+\delta t_2}\right)t_2\delta+6ct_2^2\log\left(\frac{1}{1+\delta t_2}\right)\delta^2+ \\ 6b\log\left(\frac{1}{1+\delta t_2}\right)t_2\delta^2+3bt_2^2\delta^3+2ct_2^3\delta^3 \end{array}\right) \quad (8)$$

3) Lost sales cost per cycle;

$$LS=\pi_l\left(\int_{t_1}^{t_1+t_2}\left((1-\frac{1}{1+\delta\left(t_1+t_2-t\right)})\left(a+bt\right)\right)dt\right)$$

$$LS=\pi_l\left(\begin{array}{l} a(t_1+t_2)+\frac{b}{2}(t_1+t_2)^2+\frac{c}{3}(t_1+t_2)^3-\left(\begin{array}{l}\frac{a}{\delta}+\frac{b}{\delta^2}+\frac{b(t_1+t_2)}{\delta}+\frac{c}{\delta^3}+ \\ \frac{2c(t_1+t_2)}{\delta^2}+\frac{c(t_1^2+t_2^2)}{\delta}+\frac{2ct_1t_2}{\delta}\end{array}\right)\log\frac{1}{1+\delta t_2} \\ +\frac{c}{\delta}\left(\frac{1}{2}(t_1+t_2)2-\frac{3}{2}t_1^2+t_1(t_1+t_2)+t_2(t_1+t_2)-t_1t_2\right)-\frac{ct_2}{\delta^2}-at_1-\frac{1}{2}bt_1^2-\frac{1}{3}ct_1^3 \end{array}\right) \quad (9)$$

4) Purchase cost per cycle = (purchase cost per unit) X (Order quantity in one cycle)

$$PC = C_1.Q$$

When t = 0 the level of inventory is maximum and it is denoted by IM (= I_1 (0)) then from the equation (4)

$$IM=-\frac{a}{\theta}+\frac{b}{\theta^2}-\frac{2c}{\theta^3}+e^{t_1\theta}\left(\frac{a}{\theta}+\frac{b(t_1-\frac{1}{\theta})}{\theta}+c\frac{(t_1^2-\frac{2t_1}{\theta}+\frac{2}{\theta^2})}{\theta}\right) \quad (10)$$

The maximum backordered of inventory occur at t = t_1 +t_2, then from the equation (6)

$$IB = -I_2\,(t_1+t_2)$$

$$IB=-\left(\begin{array}{l}\left(\frac{a}{\delta}+\frac{b}{\delta^2}+\frac{b(t_1+t_2)}{\delta}+\frac{c}{\delta^3}+\frac{2c(t_1+t_2)}{\delta^2}+\frac{c(t_1^2+t_2^2)}{\delta}+\frac{2ct_1t_2}{\delta}\right)\log\frac{1}{1+\delta t_2} \\ +\frac{c}{\delta}\left(\frac{1}{2}(t_1+t_2)^2-\frac{3}{2}t_1^2+t_1(t_1+t_2)+t_2(t_1+t_2)-t_1t_2\right)+\frac{ct_2}{\delta^2}+\frac{bt_2}{\delta}\end{array}\right) \quad (11)$$

Thus the order size during total time interval [0, T] is

Q = IM + IB

Now from equations (10) and (11)

$$Q=\left(\begin{array}{l}-\frac{a}{\theta}+\frac{b}{\theta^2}-\frac{2c}{\theta^3}+e^{t_1\theta}\left(\frac{a}{\theta}+\frac{b(t_1-\frac{1}{\theta})}{\theta}+c\frac{(t_1^2-\frac{2t_1}{\theta}+\frac{2}{\theta^2})}{\theta}\right)-\\ \left(\begin{array}{l}\frac{a}{\delta}+\frac{b}{\delta^2}+\frac{b(t_1+t_2)}{\delta}+\frac{c}{\delta^3}+\\ \frac{2c(t_1+t_2)}{\delta^2}+\frac{c(t_1^2+t_2^2)}{\delta}+\frac{2ct_1t_2}{\delta}\end{array}\right)\log\frac{1}{1+\delta t_2}\\ -\frac{c}{\delta}\left(\frac{1}{2}(t_1+t_2)^2-\frac{3}{2}t_1^2+t_1(t_1+t_2)+t_2(t_1+t_2)-t_1t_2\right)-\frac{ct_2}{\delta^2}-\frac{bt_2}{\delta}\end{array}\right) \tag{12}$$

Thus PC = C_1.Q

$$PC=C_1Q\left(\begin{array}{l}-\frac{a}{\theta}+\frac{b}{\theta^2}-\frac{2c}{\theta^3}+e^{t_1\theta}\left(\frac{a}{\theta}+\frac{b(t_1-\frac{1}{\theta})}{\theta}+c\frac{(t_1^2-\frac{2t_1}{\theta}+\frac{2}{\theta^2})}{\theta}\right)-\\ \left(\begin{array}{l}\frac{a}{\delta}+\frac{b}{\delta^2}+\frac{b(t_1+t_2)}{\delta}+\frac{c}{\delta^3}+\\ \frac{2c(t_1+t_2)}{\delta^2}+\frac{c(t_1^2+t_2^2)}{\delta}+\frac{2ct_1t_2}{\delta}\end{array}\right)\log\frac{1}{1+\delta t_2}\\ -\frac{c}{\delta}\left(\frac{1}{2}(t_1+t_2)^2-\frac{3}{2}t_1^2+t_1(t_1+t_2)+t_2(t_1+t_2)-t_1t_2\right)-\frac{ct_2}{\delta^2}-\frac{bt_2}{\delta}\end{array}\right) \tag{13}$$

5) Ordering Cost

OC = A (14)

Therefore the total cost per time unit is given by,

$TC(t_1,t_2)=\frac{1}{(t_1+t_2)}$ [Ordering cost + carrying cost + backordering cost + lost sale cost + purchase Cost]

$$TC(t_1,t_2)=\frac{1}{(t_1+t_2)}\text{ [OC+ IHC + BC + LS + PC]} \tag{15}$$

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$$
TC(t_1,t_2)=\frac{1}{(t_1+t_2)}\left[\begin{array}{l}
A+\left(\begin{array}{l}
-\dfrac{1}{12\theta^5}(-12\beta b\theta+24\alpha c\theta+12\beta a\theta^2-12b\theta^2\alpha-24c\beta e^{\theta t_1}\\
+4b\theta^4\beta t_1^{\,3}+3c\theta^4\beta t_1^{\,4}-12a\alpha\theta^3e^{\theta t_1}-12a\beta\theta^2e^{\theta t_1}-24c\alpha\theta e^{\theta t_1}\\
+12b\alpha\theta^2e^{\theta t_1}+12b\beta\theta e^{\theta t_1}-12c\alpha\theta^3t_1^{\,2}e^{\theta t_1}-12b\alpha\theta^3t_1e^{\theta t_1}\\
+24c\beta\theta t_1e^{\theta t_1}-12c\beta\theta^2t_1^{\,2}e^{\theta t_1}-12b\beta\theta^2t_1e^{\theta t_1}+24c\alpha\theta^2t_1e^{\theta t_1}\\
+6b\beta\theta^3t_1^{\,2}+12a\alpha\theta^4t_1+6b\alpha\theta^4t_1^{\,2}+6a\beta\theta^4t_1^{\,2}+4c\theta^4\alpha t_1^{\,3}\\
+4c\theta^3\beta t_1^{\,3}+24\beta c+12a\alpha\theta^3+12a\beta\theta^3t_1)
\end{array}\right)+\\[2ex]
\dfrac{\pi_b}{6\delta^4}\left(\begin{array}{l}
12ct_1t_2\delta^2+6c\log\left(\dfrac{1}{1+\delta t_2}\right)+6a\log\left(\dfrac{1}{1+\delta t_2}\right)\delta^2+6b\log\left(\dfrac{1}{1+\delta t_2}\right)\delta\\
+6bt_2t_1\delta^3+6bt_2\delta^2+6ct_2\delta+9ct_2^2\delta^2+6at_2\delta^3+6ct_1^2t_2\delta^3+6ct_1t_2^2\delta^3\\
+12ct_1t_2\log\left(\dfrac{1}{1+\delta t_2}\right)\delta^2+6b\log\left(\dfrac{1}{1+\delta t_2}\right)t_1\delta^2+12c\log\left(\dfrac{1}{1+\delta t_2}\right)t_1\delta\\
+6ct_1^2\log\left(\dfrac{1}{1+\delta t_2}\right)\delta^2+12c\log\left(\dfrac{1}{1+\delta t_2}\right)t_2\delta+6ct_2^2\log\left(\dfrac{1}{1+\delta t_2}\right)\delta^2+\\
6b\log\left(\dfrac{1}{1+\delta t_2}\right)t_2\delta^2+3bt_2^2\delta^3+2ct_2^3\delta^3
\end{array}\right)+\\[2ex]
\pi_l\left(\begin{array}{l}
a(t_1+t_2)+\dfrac{b}{2}(t_1+t_2)^2+\dfrac{c}{3}(t_1+t_2)^3-\left(\begin{array}{l}\dfrac{a}{\delta}+\dfrac{b}{\delta^2}+\dfrac{b(t_1+t_2)}{\delta}+\dfrac{c}{\delta^3}+\\ \dfrac{2c(t_1+t_2)}{\delta^2}+\dfrac{c(t_1^2+t_2^2)}{\delta}+\dfrac{2ct_1t_2}{\delta}\end{array}\right)\log\dfrac{1}{1+\delta t_2}\\
+\dfrac{c}{\delta}\left(\dfrac{1}{2}(t_1+t_2)2-\dfrac{3}{2}t_1^2+t_1(t_1+t_2)+t_2(t_1+t_2)-t_1t_2\right)-\dfrac{ct_2}{\delta^2}-at_1-\dfrac{1}{2}bt_1^2-\dfrac{1}{3}ct_1^3
\end{array}\right)+\\[2ex]
C_1Q\left(\begin{array}{l}
-\dfrac{a}{\theta}+\dfrac{b}{\theta^2}-\dfrac{2c}{\theta^3}+e^{t_1\theta}\left(\dfrac{a}{\theta}+\dfrac{b(t_1-\frac{1}{\theta})}{\theta}+c\dfrac{(t_1^2-\frac{2t_1}{\theta}+\frac{2}{\theta^2})}{\theta}\right)-\\
\left(\begin{array}{l}\dfrac{a}{\delta}+\dfrac{b}{\delta^2}+\dfrac{b(t_1+t_2)}{\delta}+\dfrac{c}{\delta^3}+\\ \dfrac{2c(t_1+t_2)}{\delta^2}+\dfrac{c(t_1^2+t_2^2)}{\delta}+\dfrac{2ct_1t_2}{\delta}\end{array}\right)\log\dfrac{1}{1+\delta t_2}\\
-\dfrac{c}{\delta}\left(\dfrac{1}{2}(t_1+t_2)^2-\dfrac{3}{2}t_1^2+t_1(t_1+t_2)+t_2(t_1+t_2)-t_1t_2\right)-\dfrac{ct_2}{\delta^2}-\dfrac{bt_2}{\delta}
\end{array}\right)
\end{array}\right] \quad (16)
$$

Differentiates the equations (16) with respect to t_1 and t_2 then we get

$$\frac{\partial TC}{\partial t_1} and \frac{\partial TC}{\partial t_2}$$

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To minimize the total cost $TC(t_1,t_2)$ per unit time the optimal value of t_1 and t_2 can be obtained by solving the following equations

$$\frac{\partial TC}{\partial t_1}=0 \text{ and } \frac{\partial TC}{\partial t_2}=0 \tag{17}$$

Provided equation (15) satisfies the following condition

$$\left(\frac{\partial^2 TC}{\partial t_1^2}\right)\left(\frac{\partial^2 TC}{\partial t_2^{\,2}}\right)-\left(\frac{\partial^2 TC}{\partial t_1 \partial t_2}\right)^2>0 \quad \text{and} \quad \left(\frac{\partial^2 TC}{\partial t_1^2}\right)>0 \tag{18}$$

By solving (17) the value of t_1 and t_2 can be obtained and if t_1 and t_2 satisfy the equation (18) then at these optimal values equations (16) provides minimum total inventory cost per unit time of the inventory system. Since the nature of the cost function is highly non linear thus the convexity of the function shown graphically in the next section.

5. Numerical and Graphical Illustration

Consider an inventory system with the following parameter in proper unit A=2500, α=0.5, β=0.011, C_1=4, π_b=12, π_l =15, δ=8, a=25, b=40, θ=0.005, c=20. The computer output of the program by using Maple mathematical software is t_1 = 2.72, t_2 = 0.02 and TC= 1540. i.e. the value of t_1 at which the inventory level becomes zero is 2.72 unit and shortage period is 0.03 unit.

If we plot the total cost function (16) with some values of t_1 and t_2 s.t., t_1=2.0 to 3.80 with equal interval t_2 = 0.01 to.03, fixed t_1 at 2.72 and t_2 varies from 0.01 to 0.03, fixed t_2 at 0.02 and t1 varies from 2.0 to 3.80 then we get strictly convex graph of total cost function (TC) given by the figure 2, 3 and 4 respectively.

Figure-2 (Total Cost vs. t1 and t2)

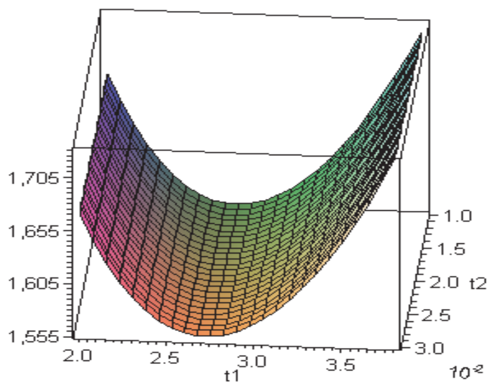

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Figure-3(Total Cost vs. t_2 at t_1=2.72)

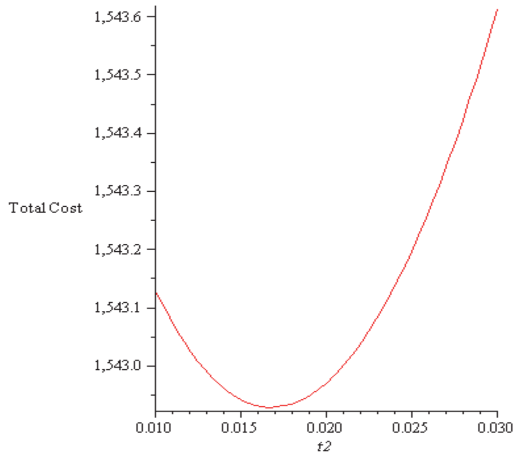
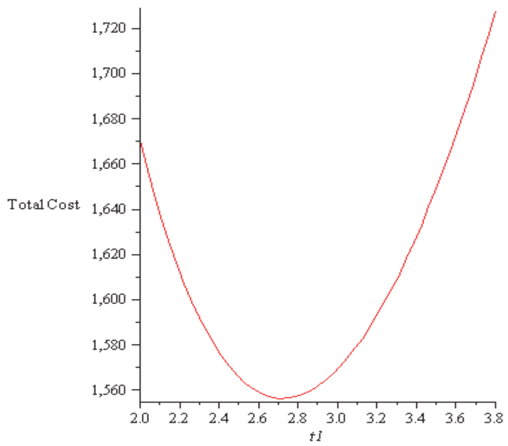


Figure-4(Total Cost vs. t_1 at t_2=0.02)



Particular cases

A. (Demand rate is linear i.e. c =0)

If we consider the following parameter in proper unit of an inventory system A=2500, α=0.5, β=0.011, C_1=4, π_b=12, π_l =15, δ=8, a=25, b=40, θ=0.005, c=0. Then the output of the program using Maple mathematical modeling software is as follows

TC=1185.34, t_1= 4.2 and t_2=0.10

If we plot the total cost function (16) with some values of t1 and t2 s.t., t1=2.0 to 7.80 with equal interval t_2 = 0.01 to.10, fixed t_1 at 2.72 and t_2 varies from 0.01 to 0.18, fixed t_2 at 0.02 and t_1 varies from 2.0 to 5.80 then we get strictly convex graph of total cost function (TC) given by the figure 5, 6 and 7 respectively.

Figure-5(Total Cost vs. t1 and t2)

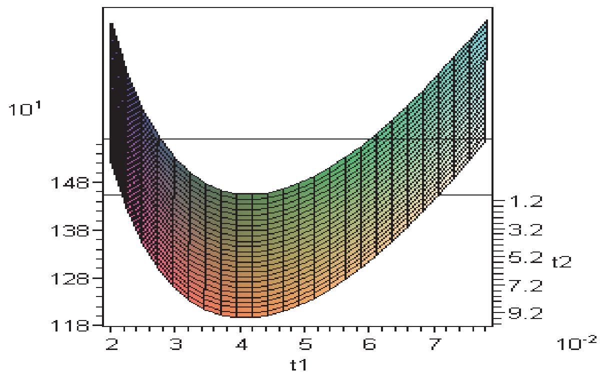


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Figure-6 (Total Cost vs. t_2 at t_1=2.72)

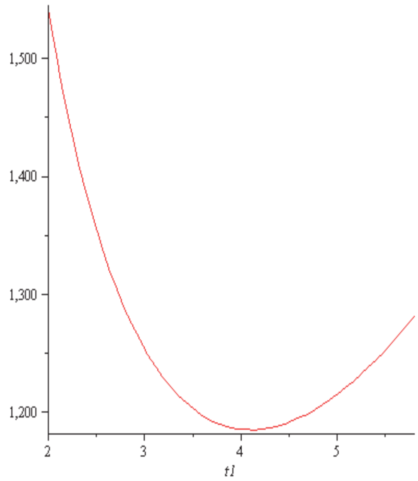
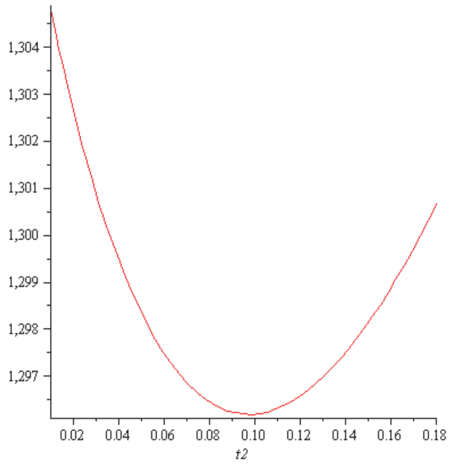


Figure-7(Total Cost vs. t_1 at t_2=0.02)



B. Demand rate is constant i.e. b=0 and c=0

If we consider the following parameter in proper unit of an inventory system A=2500, α=0.5, β=0.011, C_1=4, π_b=12, π_l =15, δ=8, a=25, b=0, θ=0.005, c=0. Then the output of the program using Maple mathematical modeling software is as follows TC=357.21, t_1= 18.87 and t_2= 0.58

If we plot the total cost function (16) with some values of t_1 and t_2 s.t., t_1=2.0 to 7.80 with equal interval t_2 = 0.01 to.10, fixed t_1 at 18.87 and t_2 varies from 0.01 to 0.92, fixed t_2 at 0.58 and t_1 varies from 2.0 to 50.80 then we get strictly convex graph of total cost function (TC) given by the figure 8, 9 and 10 respectively.

Figure-8 (Total Cost vs. t_1 and t_2)

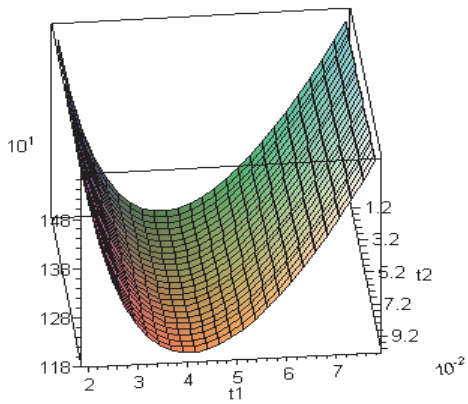


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Figure-9(Total Cost vs. t_2 at t_1=18.87)

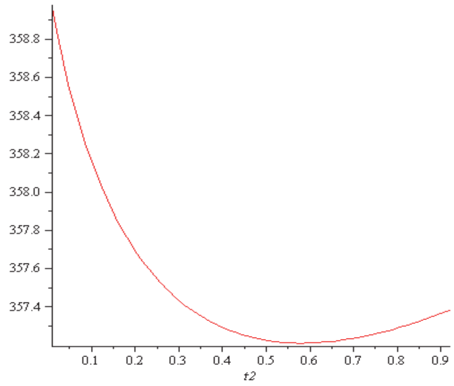
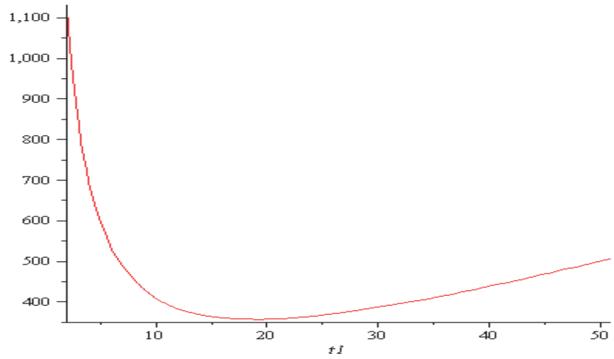


Figure-10(Total Cost vs. t_1 at t_2=0.58)



6. Conclusion and future research

This paper presents an inventory model for direct application to the business enterprises that consider the fact that the storage item is deteriorated during storage periods and in which the demand is quadratic function of time and holding cost is linear function of time and deterioration rate is constant. The assumption of quadratic demand which is useful for the items whose demand increases very rapidly, such as newly launched products in the market. This inventory model is also applicable for the situation when demand rate is linear function of time or constant with constant deterioration rate by minor change in the incremental factor b and c of quadratic function. The analytical solution of the model has given that minimizes the total inventory cost. Finally, the proposed model has been verified by a numerical and graphical analysis. The obtained results indicate the validity and stability of the model. The model is very useful in the situation in which the demand rate and holding cost is depending upon the time. This work can further be extended for other forms of demand rate, for the case of fully backlogged and for variable deterioration rate.

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