

Reflections from European examples on the teaching of modelling

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Abstract

Las recomendaciones del Parlamento Europeo y el informe Rocard invitan a desarrollar la enseñanza de la modelización; además, distintos proyectos europeos como LEMA, PRIMAS y STEAM promueven el desarrollo de recursos, preparan a profesores e investigan en modelización. Vamos a analizar, a través de ejemplos de distintos países de Europa, cómo se enseña la modelización desde la educación secundaria hasta la formación de profesores. A través de estos ejemplos observaremos niveles de determinación diferentes así como distintas cuestiones didácticas relacionadas, tanto con la praxis del aula, como con las organizaciones didácticas. El ejemplo del programa europeo LEMA sirve para ilustrar cómo se desarrolla y se implementa un curso de formación en modelización para profesores en activo. Presentaremos algunos resultados de investigaciones recientes que aportan nuevos retos sobre modelización y veremos cómo se trabaja la modelización en distintos países de Europa. En primer lugar profundizaremos sobre el contexto institucional (del global al local) dando algunos ejemplos concretos, en la segunda parte nos centraremos en uno en concreto, el proyecto LEMA, analizándolo a partir de investigaciones recientes.

Recommendations of European parliament and Rocard 's report invite to develop the teaching of modelling. The European Commission programmes like LEMA, PRIMAS and STEAM support the development of resources, training and research on the teaching of modelling. We will study European examples about the teaching of modelling, from secondary and tertiary education and from pre-service and in-service teachers training. They point the different levels of determination and the different didactic questions related to students and teachers practices and to mathematical and didactical organisations. The example of the European program LEMA illustrates a teacher training course on modelling and the difficulties to implement a teaching of modelling. We will present some recent results of research bringing challenges for the teaching of modelling. The idea of this talk is to take examples in Europa about the teaching of modelling in order to reflect on this teaching. In a first time we will browse the institutional context from global to local where the teaching of modelling takes place by giving examples from Europe. Then we develop one of these examples, the LEMA project, in order to reflect on teaching of modelling by illustrating with recent researches.

Keywords: Matemática, modelización , Europa, enseñanza, profesor, formación, institución.
Mathematic, modeling, Europe, teaching, teacher, training, institution

1 Level of determination of the society

We will use the theoretical framework of Chevallard ‘anthropological of the didactic ([10]) that makes the focus on the role of institutions. First we will consider institution at the level of the society: PISA, European Parliament and Rocard’s report for European commission. **PISA determination: mathematisation and competences**

I will first mention PISA because a lot of curricula are mentioning PISA to justify parts of their curriculum. For example in Germany the Standing Conference of the Ministers of Education and Cultural Affairs had adopted in 2003 educational standards for secondary school (Bildungsstandards) by referring explicitly to PISA ([17, p. 4]). In France, a curricular reform in 2006 ([5, p.III]) defined a Common Base of Knowledge and Skills (Socle Commun de Connaissances et de Compétences) that all pupils must progressively acquire throughout their compulsory schooling. This French reform refers explicitly to PISA. PISA assess whether 15 year-old students are able to mathematise.

PISA studies how students solve and interpret mathematical problems in varied situations inspired by real-life. PISA underlines the importance of the process of mathematisation and proposed a mathematisation cycle based on the works of Schupp (1988) and Blum (1996). PISA identifies 5 steps in the mathematisation cycle ([23, p. 107]):

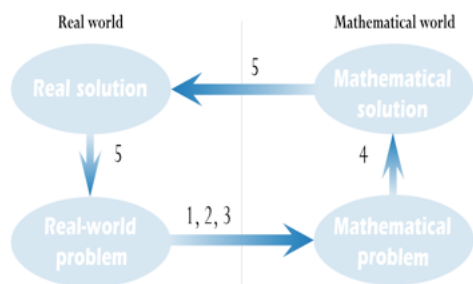


Figure 1: PISA mathematisation cycle [23].

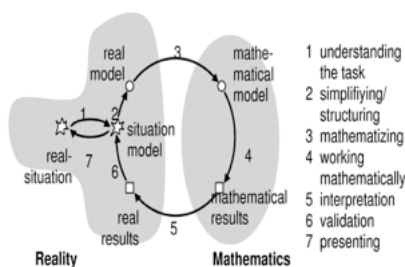


Figure 2: Mathematisation cycle from [3].

1. Starting with a problem situated in reality.
2. Organising it according to mathematical concepts and identifying the relevant mathematics involved.
3. Gradually trimming away the reality through processes such as making assumptions, generalising and formalising. These processes promote the mathematical features of the situation and transform the real-world problem into a mathematical problem that faithfully represents the situation.
4. Solving the mathematical problem.
5. Making sense of the mathematical solution in terms of the real situation, including identifying the limitations of the solution.

Other cycles of mathematisation can be proposed. For example [3] proposes a mathematisation cycle with a real model before the mathematical model and focusing in the relation between the reality and the modelled situation in a cognitive perspective.

In some countries like France mathematisation can be considered inside mathematics, what corresponds to the vertical right part of PISA mathematisation cycle ([18]).

Another important development supported by PISA is the introduction of competences inspired by the works of Niss ([21]). A lot of curricula take in account competences, for example “socle de connaissances et de compétences” in France ([5]), “competencias basicas” in Spain ([9]) or “Bildungsstandards” in Germany ([17]).

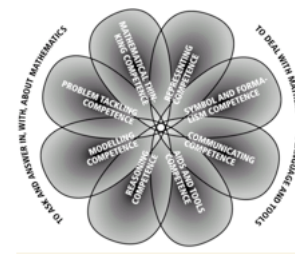


Figura 3: Competences [21].

PISA considers three competency clusters ([23, p. 115]).

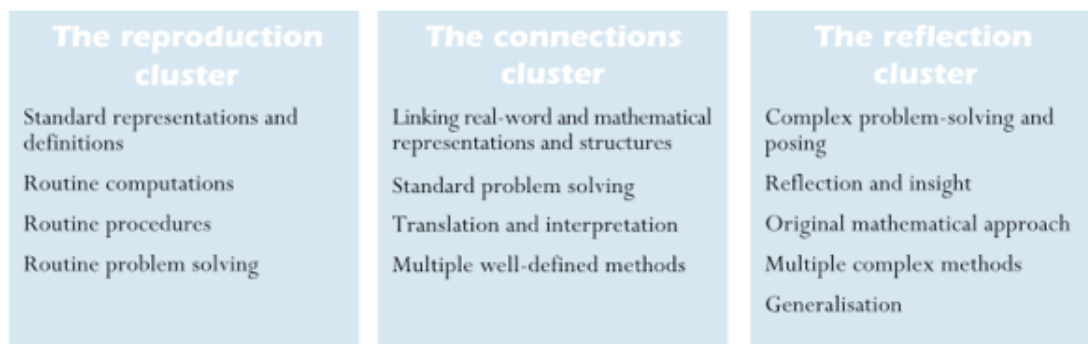


Figura 4: PISA competency clusters[23, p. 115].

PISA considers that mathematisation occurs in all competences involved in problem solving: “The process of mathematisation occurs in two different phases: horizontal mathematisation, which is the process of translating the real world into the mathematical world, and vertical mathematisation, that is, working on a problem within the mathematical world and using mathematical tools in order to solve the problem. [...] One can argue that mathematisation occurs in all competency classes because, in any contextualised problem, one needs to identify the relevant mathematics” ([22, p. 47]). It is interesting to observe that European parliament has made recommendations about the use of mathematical competence in relation with everyday situations.

European determination: mathematical competence in relation with everyday and sciences situations

In 2006 the European parliament recommends that member States use the ‘Key Competences for Lifelong Learning’ in initial education and training of young people or adults. For European ”Mathematical competence is the ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations” ([16]). Numerous curricula are explicitly referring to this recommendation, for example in France Common Base of Knowledge ([5, p.III]) or in Spain basic competences [9, p. 686]).

The European Commission has tasked a group of experts led by M. Rocard, former French prime minister, to examine how good practices could develop young people’s interest in science studies and to identify necessary conditions to help this development. This report recommends introduction of inquiry based approach (ISBE): “Improvements in science education should be brought about through new forms of pedagogy: the introduction of inquiry-based approaches in schools, actions for teachers training to IBSE, and the development of teacher’s networks

should be actively promoted and supported” ([27, p2]). “By definition, inquiry is the intentional process of diagnosing problems, critiquing experiments, and distinguishing alternatives, planning investigations, researching conjectures, searching for information, constructing models, debating with peers, and forming coherent arguments”. A clear relation between ISBE and modelling is done.

The consequence of all these institutional recommendations is the development of numerous European programmes concerning education and research and providing resources and networks that could support the development of modelling. Let us examine some of these programmes.

POLLEN ([25]) was an European programme from 2006 to 2009 that created a net of twelve cities throughout Europe. This net provides material, methodological and pedagogical supports to build on a local level initiatives developing science education at primary school. Examples illustrates inquiry based approach. The link with mathematics is not always evident in the provided resources but a didactic work could create from this resources interesting activities relating mathematics and sciences. LEMA ([20]) was an European Comenius project in which mathematics educators from six countries worked to produce materials to support teachers professional development. LEMA means Learning and Education in and through Modelling and Applications. The overall aim of the project was to facilitate a change in teacher’s classroom practices so as to include mathematical modelling activities. Relevant modelling tasks to use with pupils is necessary but not sufficient. To provide further support LEMA developed a professional development programme for teachers, together with supporting materials. This was the result of three years work (from 2006 to 2009) that involved a cycle of piloting and improvement within the project. S-TEAM (Science teacher education advanced methods) was from 2009 to 2012 a 26-partners programme to improve science teaching and learning through Europe by creating resources and teachers professional development on these subjects ([31]). COMPASS ([15]) is a project worked in the period 2009-2011 to provide tasks for teachers to develop interdisciplinary approaches that bring together mathematics and science. PRIMAS ([26]) is a project from 2010 to 2013 to promote inquiry-based learning in mathematics and science at both primary and secondary levels across Europe. Various resources and support measures have been developed and made available to teachers, parents and pupils. It is difficult to know the impact of these various programmes but they illustrate the political will to change education different subjects as modelling, inquiry based approach, connection between sciences and mathematics, science education. All these subjects can be related to modelling. A lot of problems of sciences take their origin in the real world. Sciences develop model to explain solution for these problems. And these scientific models could be based on a mathematic model. The previous Blum and Leiss mathematisation cycle can be considered as an example of cycle integrating these two models. We will observe now some examples at the level of schoolsystem and of the pedagogy in France and in Germany.

2 Level of determination of the school: French examples

France is a centralised country and the same official texts edited by the Ministry of National Education (MEN) describe the curriculum and are applied everywhere.

Exploratory teaching (*enseignement exploratoire*)

For example, in grade 10 “classe de seconde” (15-16 years old), there is from 2010 an optional course ” (exploratory teaching) on scientific methods and practices ([7, p. 1]). This course takes one and a half hour per week in student’s time-tables. It allows them to explore different

areas of mathematics, physics and chemistry, life sciences and earth and engineering sciences. The following skills are developed: to use knowledge and complete knowledge; to learn, search, extract and organize useful information (written, oral, observable, digital); to reason, argue, make a scientific approach, demonstrate; to communicate using a language and tools. We recognize modelling skills suggested by PISA but formulated in an other way.

Supervised personal work (travaux personnels encadrés TPE)

From 2000, during grade 11 (16-17 years old), students of the scientific branch have to undertake a supervised project, called TPE ([8]). Over eighteen weeks, small groups of students work collectively on a project. They choose a problematic subject related to national topics; they use varied resources. They have to connect two disciplines including one which is essential to the students orientation (for example mathematic in the scientific branch). Examples of subject are: How can we use satellite images to refine forecasted monsoons? Modification of food is it progress?). The project is supervised by teachers of the relevant disciplines with two hours per week in the students timetable. Assessment considers the development of the project, written and oral presentations, and is part of the final mark of examination (baccalauréat) for entering university. For 2011, an example of national research axis for scientific branch is: basic and applied sciences as they relate to the technical achievements. Examples of subjects are: The historical context of the evolution of science and technology and their relationships. Understanding of phenomena, prior achievements techniques. Mathematics at work in the large technical projects. Modeling and simulation. Control of materials at the service of new achievements techniques.

Mono or pluri-disciplinary teachers

These pluridisciplinary school organisations (Exploratory teaching, supervised personal work) can easier supports for modelling activities where pluridisciplinary occurs, than one subject mathematic lesson. It would be interesting to investigate to study if the fact that the teacher are one ou multi-disciplines teachers plays a rôle in the development of modelling at school. In France primary school teacher are multi-disciplines teachers but mathematic teacher at secondary school are one subject teacher. On the contrary, in Germany, primary and secondary school teachers are multi-disciplines teachers. ([28]) shows for Canada that elementary teacher don't use so much pluridisciplinary activities involving mathematic.

3 Level of pedagogy: examples from France and Germany

At the level of pedagogy we consider the determination not directly related to one discipline but that could be related to several or all disciplines.

Theme of convergence in French low secondary school

In France, the "college" takes place after primary school from grade 6 to to grade 9 (from 11 to 15 years old) in a comprehensive school. From 2008 the «college» syllabus proposes a common introduction for all the scientific subjects and defines ([6, p. 5]) different «themes of convergence » to be worked together by different disciplines and support a common inquiry based approach, what fits very well with modelling activities. Examples of themes are: importance of statistics thought on the scientific view of the world, sustainable development, energy... The syllabus insists on the differences and on the similarities between non mathematical sciences and mathematics in the inquiry based approach, specially about the difference in the validation step and in the meaning of hypothesis: "the inquiry based approach has similarities between

its application to the field of experimental sciences and to the mathematical one [...] A comprehensive scientific education must make students aware of both the proximity of these steps (problemsolving, formulation respectively of explanatory hypotheses and of conjectures) and specific features of each, particularly in regard to validation, by experimenting on one side, by the demonstration on the other side (Ibidem. p.4). The importance of modelling in the themes of convergence is confirmed: “Mathematics provides powerful tools for modeling phenomena and predict results, particularly in the field of experimental sciences and technology, allowing the expression and development of many elements of knowledge” ([6, p. 2]).

Sinus programme in Germany to change pedagogical approach

From 2003 Sinus-transfer is a programme applied in Germany by regional school education authorities in order to change the pedagogical approach on science and mathematic teaching. It emphasizes scientific inquiry and experimental approaches. The main tools used in this programme are: teacher development, production of resources and networks between schools and teachers. The impact of this programme is very positive, on student attainment, on weaker students and on teachers. In this project students should experiment, observe, discover, conjecture, explain and justify, what are competences related to modelling. Explicit reference to PISA is made and flexibility in applying mathematical concepts and translating insights into mathematical content (modeling) has to be involved.

4 Levels of determination of mathematic as discipline

In France and Baden-Württemberg (Germany) ”in the curriculum of primary school, modelling is not explicitly knowledge to be taught but it can be implicitly considered as knowledge to be taught as propedeutic to the secondary school curriculum (Baden-Württemberg) or as a part of problem solving (France). The consequence is that modelling is not explicitly a study theme in the textbooks. Nevertheless modelling tasks appear in textbooks involving varied domains of mathematic world and of real word. Furthermore teaching tasks appear that are not modelling tasks but that support achieving partial competencies as prerequisite of work on modelling task, what shows that modelling is a taught object. Some mathematic textbooks plan, through the school year, the teaching of real world knowledge and mathematical knowledge and their articulation” ([14, p. 567]). For secondary school in France and Spain ([12]) have shown that modelling is designated to be taught in the mathematical syllabus but “in contrast with Germany, where modelling is one of the seven core competences of the secondary mathematics curriculum, in neither France nor Spain is modelling so explicitly defined. Official texts discuss modelling both explicitly and implicitly but it is not always clear if students are expected to apply a given model or construct a model in order to solve a problem. However in the French texts is mention of the part of the modelling cycle where the model is built. Indeed, there are several resources from the French Ministry in which can be found classroom tasks where models have to be built, like for example in probability”.

5 Local levels of determination for setting the model and validating

To illustrate local levels of determination (domain, sector, theme, subject) related to mathematic we will consider modelling at the level of determination of a task proposed to pupils.

The mathematisation cycle adopted by LEMA European project is inspired from PISA cycle and integrates five competences. Four competences are corresponding to parts of the cycle: setting up the model, working accurately (in the mathematic world), interpreting the mathematic solution in a real solution, validating and reflecting. A fifth competence is a transversal one: reporting the work. Let us consider a task proposed by ([20]): “signing against the law”.

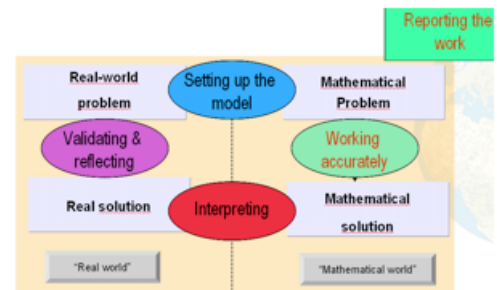


Figure 5: Lema mathematisation cycle [20].

We will observe two solutions proposed by French pupils from grade 9 ([24]).

Recently, the 25th of April of 2006, the Spanish party in the opposition presented in the congress 4.000.000 signatures against a new law promoted by the government.

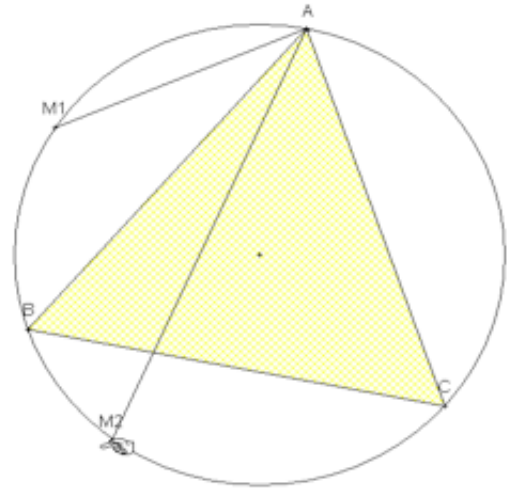


All Spanish newspapers published pictures with the big boxes and the 10 vans needed to transport the sheets of paper to the congress. Do you think there was a political intention behind this staging or all these boxes and vans were really necessary to carry the 4000000 signatures?

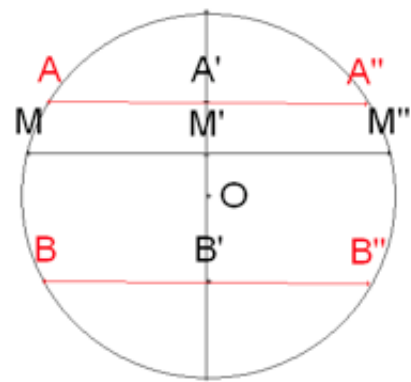
In a first solution a group of pupils estimate the amount of paper sheets corresponding to 4000000 signatures. Then they estimate the volume needed to carry these signatures and check that the available volume provided by the vans is sufficient. The second group estimate the weight of the amount of papers corresponding to the number of signatures and conclude that the maximal weight that 10 vans can carried is not sufficient to carry the signatures. The first model is based on the volume and the second one on the weight. This example illustrates that the central point in modelling is to set up a model. Most of the exercises provided in textbooks imposed the models and the exercices are only applications of the model, working only the competences working accurately, interpreting, validating and reflecting. The previous examples illustrates that several models are possible that could bring contradictory answers, what could be a didactic problem. We can argue that, even if it is possible with the volume, if it is not possible with the weight, the vans wouldn't be able to deliver the signatures. But when you look for a solution, you have no warranty that you will control all the parameters. In our previous example, if only the solution using the volume model is found, this solution will be validated. In this case, the validation is under the conditions of this model. This problem concerns the validation and the reflection about the found solution. What extra-mathematical arguments can help to validate? Here we have a pragmatic argument: even if it is possible with the volume, it would not be possible with the weight. The same problem of validation can occur in an intra-mathematical mathematisation as the following task illustrates.

Bertrand paradox

An equilateral triangle is inscribed in a circle. We choose at random a chord of the circle. What is the probability than a side of the triangle is shorter than the chosen chord? Different models of the choice at random can be proposed. In a first model, we fix a point A on the circle. To choose at random a chord on the circle means to choose a point M on the circle and to consider the chord $[AM]$. The chord $[AM]$ is longer than the side of the equilateral triangle ABC if M is chosen on the arc joining B and C . This arc has a length equal to the third of the length of the circle. With this model the probability is equal to one-third. With another model, we consider a diameter of the circle of direction $(A'B')$.



To choose at random a chord $[MM'']$ on this circle means to consider this chord perpendicular to this diameter and to choose the middle M' of this chord on this diameter. The chords $[AA'']$ and $[BB'']$ are perpendicular to this diameter and are sides of equilateral triangle inscribed in the circle. The middles A' and B' of these chords are positioned at quarter or three quarters of the diameter. The only chords $[MM'']$ longer than the side of an equilateral triangle inscribed in the circle must have their middle M' chosen between A' and B' on the diameter.



The length of $[A'B']$ is half of the length of the diameter. With this model the probability is equal to one-half.

In both examples, modelling in real world and modelling in intra-mathematical world, we see that setting the model and validating the model are problematic competences. Different models could bring different solutions. The validation of the models in the relation with real world can use extra-mathematical arguments (the maximal transport weight of a van). The didactical question is: how this extra-mathematical knowledge and validations are brought in the class? How are managed the articulation between extra-mathematical validation and mathematical one? What is the effect on the didactical contract? In the mathematical world the validation is based on conditional reasoning and on non contradiction. In the case of Bertrand paradox, there is no contradiction: two different models bring two different solutions.

Contradiction between data, hypotheses and model

Sometimes the contradiction can occur in the class without be planned by the teacher. Let us present a task from LEMA project: the giant task. The task was proposed to a group of French CM1 (grade 5: 10-11 years old). What is the approximate size of silhouette, which can see only a foot? This photo was taken in an amusement park. One solution proposed by pupils is the following.



On the photo we measures: for the man with blue jacket, height about 7.5cm , shoe length about 1cm ; for the giant: shoe length about 9cm . In the reality we assume: man's shoe about 30cm and man's height about 180cm . We assume that the ratio between giant's shoe length and man's shoe length on the photo are the same for all the corresponding length in the reality. The ratio giant's shoe/ man's shoe is $9/1 = 9$ on the photo. In the reality a man is about 180cm height. We assume that a man is a reduction of a giant. It means that in the reality giant's height is about $9 \times 180\text{cm} = 1620\text{cm}$.

In fact this solution is not valid because there is a contradiction between model, hypotheses and data. Here the model is that a photo is a reduction of the reality and a man is a reduction of a giant (proportionality model). By assuming that the photo is a reduction of the reality, the corresponding ratio between photo and the reality are the same. With the assumption on data we get a ratio in the reality between man's height and man's shoe length: $180/30 = 6$. The ratio on the photo between man's height and man's shoe length is $7.5/1 = 7.5$. We get a different ratio between man's height and man's shoe length on the photo and in the reality. It is a contradiction with the model assuming that photo is a reduction with conservation of ration between photo and reality (proportionality model). We can discuss that the assumption on the length of man's shoe is not necessary. When we choose an assumption in order to find a solution, we don't know if this assumption will be successful. Without supplementary assumption on man's shoe length in the reality (or on man's height in the reality) pupils haven't found a solution. In the experience developed in the grade 5 class, pupils have developed a model in contradiction with data and hypotheses. We are in the Lakatos' context ([19]): so long we don't find a refutation, the solution is considered as correct. The validity of the real solution keeps so long the real world or the mathematical world don't give a refutation. It is a general characteristics of laws in experimental sciences.

All these examples show how the task proposed to pupils determine how problematic setting a model and validating a solution are. We will conclude now with some questions and propositions related to teachers' training, resources and teaching.

6 Conclusion

We have shown the importance of institutional approach by pointing different levels of determination of teaching of modelling.

At global level, European institutions encourage the development of modelling in curriculum and of ressources available for teachers and schools. ([13]) have shown teacher's interest for in-service training to teach modelling. When modelling is a new topic, teachers seem focusing on

pupil's praxeology: what task to offer to pupils? What solutions can be found by pupils? What difficulties they can meet? When modelling is part of the curriculum, teachers seem focusing on teachers praxeology: when to introduce the topic? What planning in the curriculum? How to assess? Lema project offers a response to both situations with an in-service training course offering five modules with the following contents: What is modelling and why modelling? How to produce and analyse modelling? How to teach modelling? How to assess modeling? How to reflect on modelling teaching?

At local level, double transposition is pointed: "What knowledges of real world and of mathematical world have to be transposed? What techniques, justifications and validations from both worlds have to be used? How these different knowledges, techniques, justifications and validations are articulated and interfere between the two worlds? What effects on teacher's practice, on pupil's learning and on class didactical contract have these articulations and interferences?" ([11]).

A recent research based on implementation of giant task in French classes have shown the following results. The teachers are too much centered on the solution of the giant task with a pre-built model based on the proportionality. They don't perceive the necessity of a work on the modelling competences to question the model and the validation of the model and of the solution. "It seems that for these teachers the praxeologies of modelling are not identified as useful knowledge to solve the problem" ([32, p. 33]). Adjage et al. ([1]) studied a long teaching sequence in a primary school class based on giant task and shows that sharing and revising peer's writings, particularly through a practice mobilizing the specific resources of writing, helps students to develop solutions involving a modelling process. This results show how important initial and in-service training on modelling is in order to improve the teaching of modelling. Further research has to investigate the learning of modelling taking in account the different levels of determination.

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